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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2018

Second Semester

Faculty of Science

Branch I (A) : Mathematics

MT 02 C07—ADVANCED TOPOLOGY

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Let c_i be a closed subset of a space X_i for $i \in I$ show that $\prod_{i \in I} c_i$ is a closed subset of $\prod_{i \in I} X_i$ w.r. to the product topology.
2. List two productive and four non-productive properties.
3. Characterise evaluation function.
4. Explain : (a) Evaluation Map ; and (b) Pseudo-metric space.
5. List the properties of the binary relation "a follows b".
6. Define filter and ultrafilter with examples.
7. Define sequentially compact space. Give example to show that compactness does not imply sequential compactness.
8. Justify : Alexandorff compactification is a foremost example of compactification. Also state its advantage.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. If the product is non-empty, then each co-ordinate space is embeddable in it—Prove.
10. Let Y be any topological space and let I, J be any sets. Prove that the process $\left(Y^{I}\right)^{J}$ and $Y^{I \times J}$ are homeomorphic to each other.

Turn over

11. Obtain a condition under which the evaluation function is one-one. State the definition used in the proof.

12. A topological space is Hausdorff iff limits of all nets in it is unique—Prove.

13. Prove that in an indiscrete space, every net converge to every point and that this property characterises indiscrete space.

14. Characterise continuity in terms of convergence of nets.

15. Show that all three forms of compactness are equivalent in a second countable space.

16. Prove : Every locally compact Hausdorff space is a Tychonoff space.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Establish Tietze characterisation of normality.

18. (a) Prove a product of topological spaces is completely regular iff each co-ordinate space is so.

    (b) Define binormal space. Prove that every binormal space is normal.

19. (a) Prove that a topological space in a Tychonoff space iff it is embedable into a cube.

    (b) Define embedding. State all the required results and prove embedding lemma.

20. (a) Characterise those families of sets which can be bases for filters.

    (b) Prove that the intersection of any family of filters on a set is again a filter on that set.

21. (a) Show that an ultrafilter converges to a point iff that point is a cluster point of it.

    (b) Prove that a topological space is compact iff every ultrafilter in it is convergent.

22. Prove that every countably compact metric space is second countable. Show also that every continuous. real-valued function on a countably compact space is bounded and attains its extrema.

$(3 \times 5 = 15)$